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Seat No.	

B.Sc. (Part-III) (Semester-VI) (CBCS)
Examination, March - 2023
MATHEMATICS
Linear Algebra (Paper-XIV)
Sub. Code : 81663

Day and Date : Friday, 02 - 06 - 2023 **Total Marks : 40**

Time : 10.30 a.m. to 12.30 p.m.

Instructions : **1) All questions are compulsory.**
2) Figures to the right indicate full marks.
3) Use of non-programmable calculators is allowed.

Q1) Select the correct alternative for each of the following. **[8]**

a) Let $S = \{(-1,0,1),(2,1,4)\}$. The value of k for which the vector $(3k + 2,3,10)$ belongs to the linear span of S is ____.

i) 2 ii) –2

iii) 8 iv) 3

b) Which of the following is incorrect?

i) A basis of a vector space is a maximal linearly independent set.

ii) A minimal generating subset of a vector space V is a basis for V.

iii) Any two bases of a F.D.V.S. have same number of vectors

iv) If dim $V = n$, then any $n + 1$ vectors in V are linearly independent.

c) If $T:U \rightarrow V$ is a linear transformation such that dim $U = 4$ and nullity $T=2$ then rank of T is _____.

i) 1 ii) 2

iii) 0 iv) 4

P.T.O.

d) If $T:R^2 \rightarrow R^2$ and $S:R^2 \rightarrow R^3$ defined by $T(x,y) = (y,x)$ and $S(x,y) = (x+y, x-y, y)$ then $ST(x,y) =$ _____.

i) $(y + x, y - x, x)$ ii) $(x - y, x + y, x)$

iii) $(x - y, x + y, y)$ iv) $(y + 2x, y - x, x)$

e) If V is an inner product space and $u, v \in V$ such that u is orthogonal to v then _____.

i) $\|u+v\|^2 = 0$ ii) $\|u+v\|^2 = \|u\|^2 + \|v\|^2$

iii) $\|u+v\|^2 \le \|u\|^2 - \|v\|^2$ iv) $\|u+v\|^2 \ge \|u\|^2 + \|v\|^2$

f) If $\{w_1, w_2, \ldots\ldots, w_n\}$ is an orthonormal set in an inner product space V, then

$$\sum_{i=1}^{n} \left|\langle w_i, v\rangle\right|^2 \le \|v\|^2 \text{ for all } v \in V$$

This property is known as ______.

i) Sylvester's law ii) Cauchy – Schwarz inequality

iii) Triangle inequality iv) Bessel's inequality

g) If $T(1,1) = (2,2)$ then______is an eigen value of T.

i) 0 ii) 1

iii) 2 iv) 3

h) The characteristic polynomial of the matrix $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ is _______.

i) $x^2 - 2x + 3$ ii) $x^2 + 3x - 10$

iii) $x^2 - 3x$ iv) $x^2 - 3x - 10$

Q2) Solve any two: **[16]**

a) If V is a F.D.V.S. and $\{v_1, v_2, v_3, ..., v_r\}$ is a Linearly independent subset of V, then show that it can be extended to form a basis of V.

b) Let V be an inner product space. Then prove that $|(u,v)| \leq \|u\| \cdot \|v\|$ for all $u, v \in V$.

c) Let V and W be two vector spaces over F. Let $\{v_1, v_2, ..., v_n\}$ be a basis of V and $w_1, w_2, ..., w_n$ be any vectors in W. Then prove that there exists a unique linear transformation $T: V \to W$ such that $T(v_i) = w_i$, $i = 1, 2, ..., n$.

Q3) Solve any four: **[16]**

a) Determine whether or not $W = \{(a,b,c) \in \mathbb{R}^3 : b = a^2\}$ is a subspace of $\mathbb{R}^3$.

b) If $T : R^3 \to R^3$ is defined as $T(x,y,z) = (x, x+y, x+y+z)$, then show that T is a linear transformation.

c) If $T: V \to U$ is a linear transformation then prove that Ker $T = \{0\}$ if and only if T is one-one.

d) Let V be an inner product space. Then prove that $\|x+y\| \leq \|x\| + \|y\|$, for all $x, y \in V$.

e) Find eigen values of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$.

f) Let P_2 be the inner product space of polynomials of degree at most 2 with respect to the inner product defined as:

$$(p, q) = \int_{-1}^{1} p(x)q(x)dx \quad \forall p, q \in P_2$$

Show that $p = x$ and $q = x^2$ are orthogonal in P_2. Find $\|p+q\|^2$.

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Seat No.	

B.Sc. (Part - III) (Semester - VI) (CBCS) Examination, March - 2023

MATHEMATICS (Paper - XVI)
Discrete Mathematics
Sub. Code : 81665

Day and Date : Monday, 05 - 06 - 2023 **Total Marks : 40**

Time : 10.30 a.m. to 12.30 p.m.

Instructions : 1) All questions are compulsory.

2) Figures to the right indicate full marks.

Q1) **Select the correct alternative for each of the following :** **[8]**

i) If p & q are two statements then $q \rightarrow p$ is called _____ of $p \rightarrow q$.

a) inverse

b) converse

c) contrapositive

d) none of these

ii) The binary addition $1101_2 + 1011_2 =$ _____.

a) 11010_2

b) 10110_2

c) 11000_2

d) 11001_2

iii) Multi graph is a graph which contains _____.

a) Parallel edges but no loops

b) Loops but no parallel edges

c) Parallel edges & loops

d) No parallel edges & no loops

P.T.O.

iv) The kind of inference $p \wedge q \therefore q$ is called _____.

a) Generalization

b) Modus Tollens

c) Specialization

d) Contradiction Rule

v) Every complete graph on 'n' vertices is an _____.

a) (n – 1)-regular graph

b) n-regular graph

c) (n + 1)-regular graph

d) $n/2$-regular graph

vi) The maximum degree of any vertex in a simple graph with n vertices is _____.

a) $n - 1$

b) $n + 1$

c) $2n - 1$

d) n

vii) A vertex of degree one in any tree is called _____.

a) Internal Vertex

b) Leaf

c) Forest

d) Isolated Vertex

viii) Which of the following is not a proposition?

a) Is mathematical boring?

b) Man landed on the sun last year

c) Diamond is harder than graphite

d) He finished his work and went away

Q2) Attempt any two of the following : **[16]**

a) Define Converse and Inverse of conditional statement and write the converse and inverse of given statement 'If today is New Year's Eve, then tomorrow is January'.

b) Define Degree of Vertex and show that number of odd degree vertices in any graph is always even.

c) Define 'Tree' and prove that for any positive integer, if G is a Connected graph with n vertices and $n - 1$ edges, then G is a tree.

Q3) Attempt any four of the following : **[16]**

a) Assume x is real number and use De Morgan's laws to write the negation of $-2 < x < 7$.

b) Prepare truth table for $(p \wedge q) \wedge \sim r$.

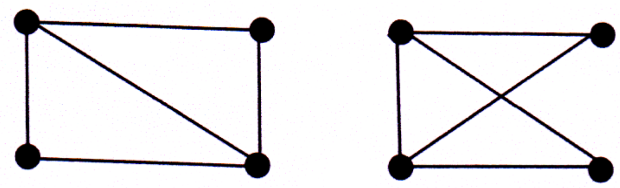
c) Using truth table show that $(p \oplus q) \oplus r \equiv p \oplus (q \oplus r)$.

d) Evaluate

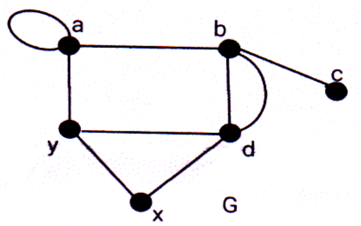
i) $101001_2 + 10011_2$

ii) $101101_2 - 10011_2$

e) Check whether the following graphs are isomorphic or not.



f) Verify Handshaking Lemma for the following graph.



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